SOLVING COMPUTATIONALLY INTENSIVE INVERSE PROBLEMS IN ELASTICITY IN 3D

Computational Mechanics Lab
MECHANICAL ENGINEERING

Ping Luo[2], Baik Jin Kim[1], Yue Mei[3], Sevan Goenezen[1]

[1]Department of Mechanical Engineering, Texas A&M University,
[2]High Performance Research Computing, Texas A&M University,
[3]Zienkiewicz Centre for Computational Engineering, Swansea University, UK

TEXAS A&M UNIVERSITY

ABSTRACT

- Mapping the non-homogeneous mechanical properties using force and surface displacement data and solving an inverse problem in elasticity.
- Surface displacements can be measured with regular digital cameras, also known as digital image correlation systems (DIC). Sets of images are recorded before and after applying a load to the specimen and processed with a DIC software to compute surface displacements.

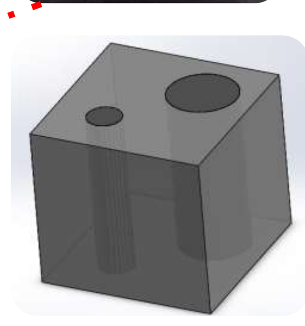
BACKGROUND

Potential application to:

- Characterize mechanical properties of additively manufacturing materials.
- Map the pore density of additively manufactured materials.
- Detect cracks in engineering materials by mapping local changes in stiffness.
- Characterize heterogeneous properties of engineered tissues and biological materials, including soft tissues.
- Detect breast tumors and cancerous skin lesions from stiffness contrasts.

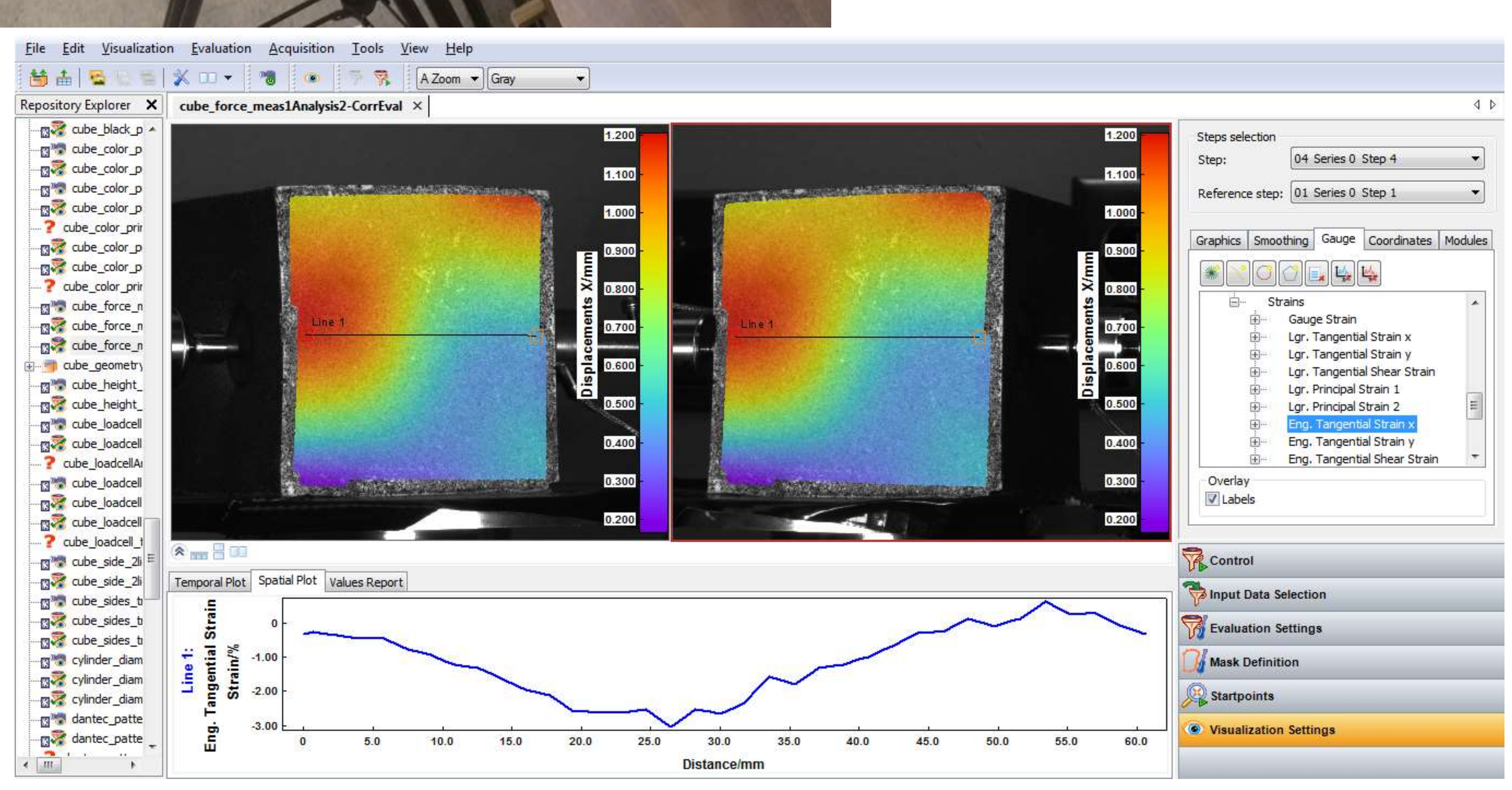
METHODOLOGY

Speckle Pattern shown on the surface of the cube



Cube with two cylindrical stiff inclusions

- Displacements are measured using a DIC system with two 5 Mega Pixel CC camera to measure displacements with 0.01 pixel accuracy.



- Data is analyzed using Instra4D

NO PRIOR ASSUMPTIONS about the material properties are being made.

In-house developed Inverse Solvers → Reconstructed Domain

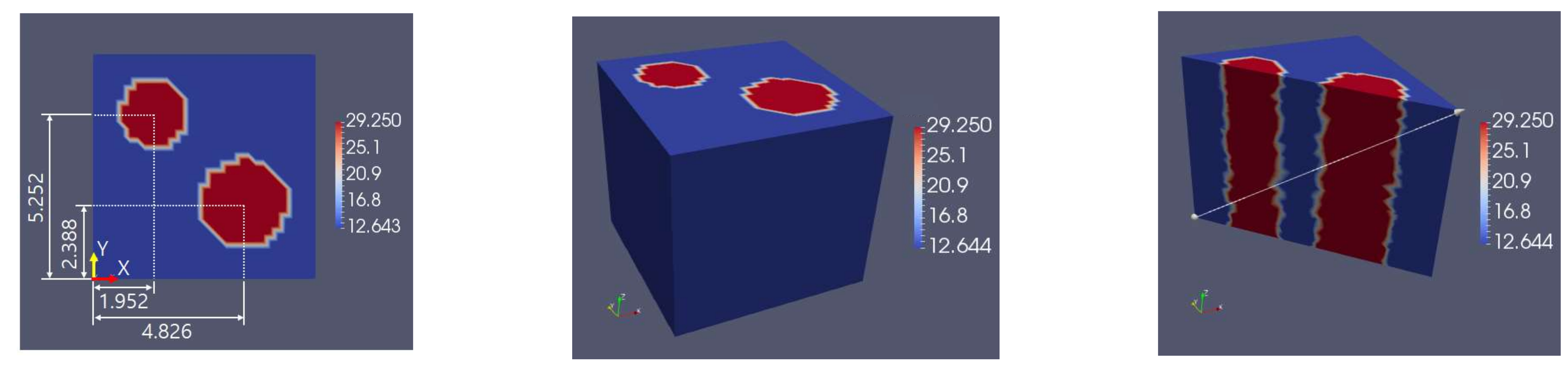
INVERSE PROBLEM

- Solving a constrained optimization problem for mechanical property distribution using a gradient basted optimization scheme (BFGS).
- Constraint of the problem is the equilibrium equation solved via FEM.
- Computation is efficiently solved using adjoint equations for multiple measurements and parallelized with Message Passing Interface (MPI).
- Computations are carried out on HPRC clusters Ada and Terra.

$$\pi = \frac{1}{2}\sum_{i=1}^{N}\left[\sum_{e=1}^{N_n}\int_{\Omega_e}\left(\sum_{j=1}^{n_e}(w_j^e)_i\,\psi_j^e(\mathbf{x})\left\{(\mathbf{u}_e^i)_{comp}-(\mathbf{u}_e^i)_{meas}\right\}_j\right)^2 d\Omega\right]+\frac{\alpha}{2}\sum_{e=1}^{N_n}\int_{\Omega_e}\sqrt{\sum_{j=1}^{n_e}|\mu_i^e\,\nabla\,\psi_j^e(\mathbf{x})|^2+c_0^2}\,d\Omega$$

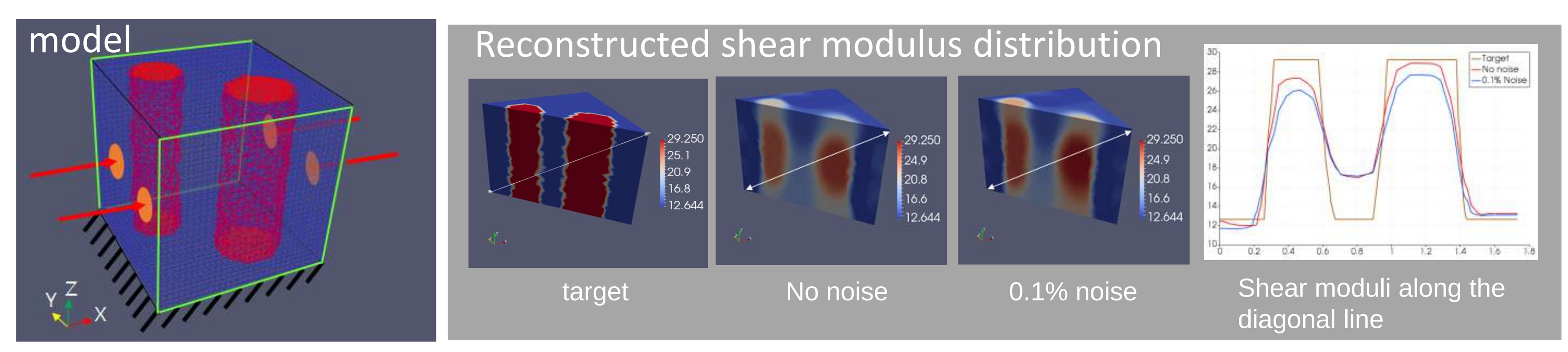
π: objective function, $(\mathbf{u}_e^i)_{comp}$: computed displacement, $(\mathbf{u}_e^i)_{meas}$: measured displacement, μ_i^e: shear modulus, $\boldsymbol{w}_j^e$: weight (0 for interior nodes & 1 for boundary nodes), ψ_j^e: shape function, α: regularization factor, $c_0 = 0.01$, N: the number of measured boundary displacement data sets, $\boldsymbol{N}_n$: the total number of elements, $\boldsymbol{n}_e$: local node number

3D PROBLEM DOMAIN

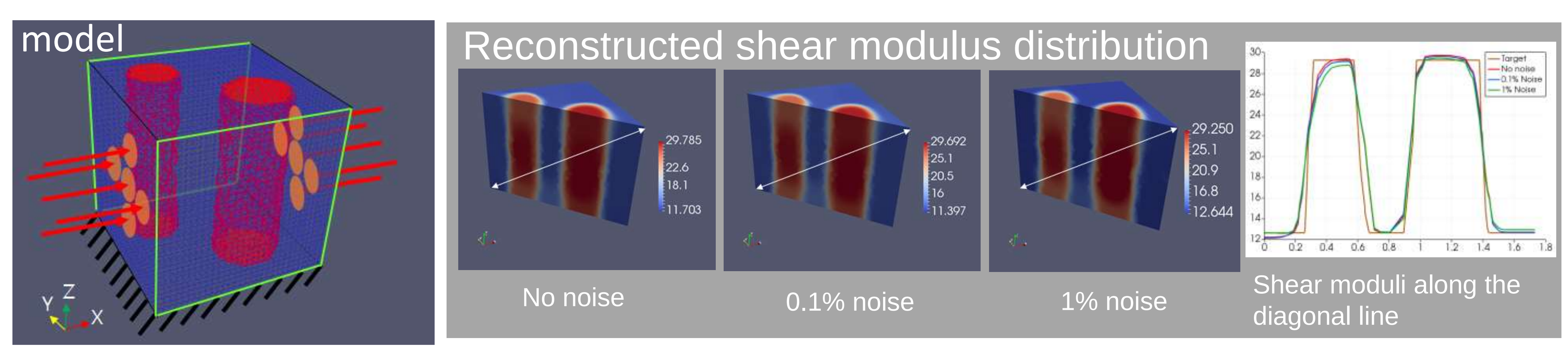


- Composite cube of size 7.1cm×7.1cm×7.1cm with two cylindrical inclusions of radius 1.09cm and 1.41cm respectively.
- Mesh composed of 20738 nodes and 108567 incompressible, linear elastic elements is used.
- Target shear modulus distribution: background is 12.644 kPa; inclusion is 29.295 kPa

SIMULATIONS WITH NO DEVIATION OF FORCE LOCATIONS

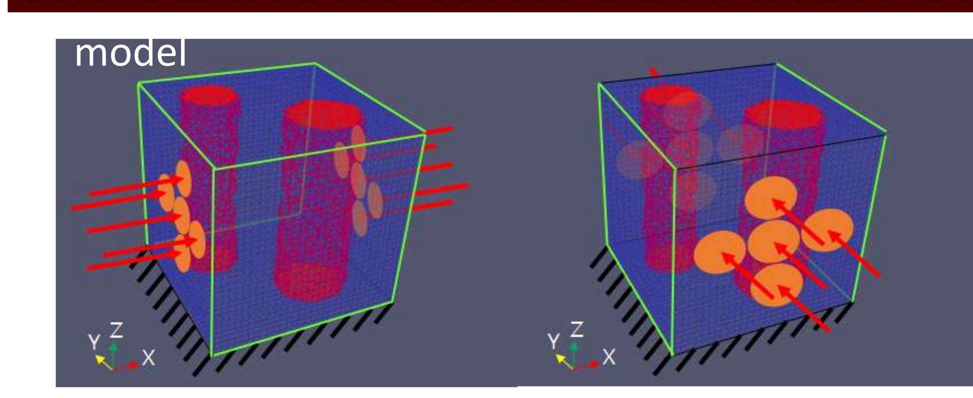


- Boundary conditions : fix the bottom surface; Apply force boundary conditions to the left and right surfaces.
- Simulate 2 boundary value problems to create 2 displacement fields.
- Minimize displacements only on the front and back surfaces

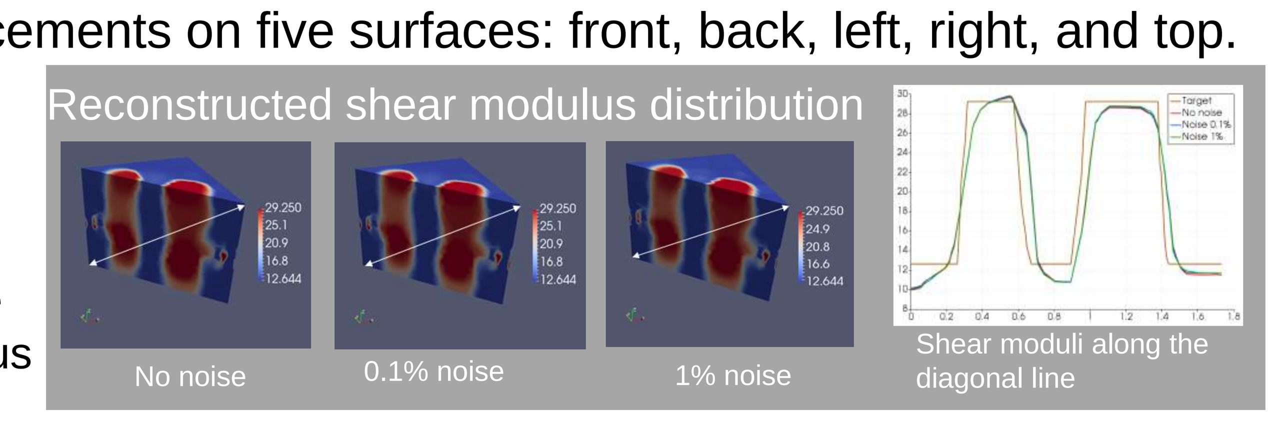


- Boundary conditions : fix the bottom surface; Apply force boundary conditions to the left and right surfaces.
- Simulate 5 boundary value problems to create 5 displacement fields.
- Minimize displacements only on the front and back surfaces

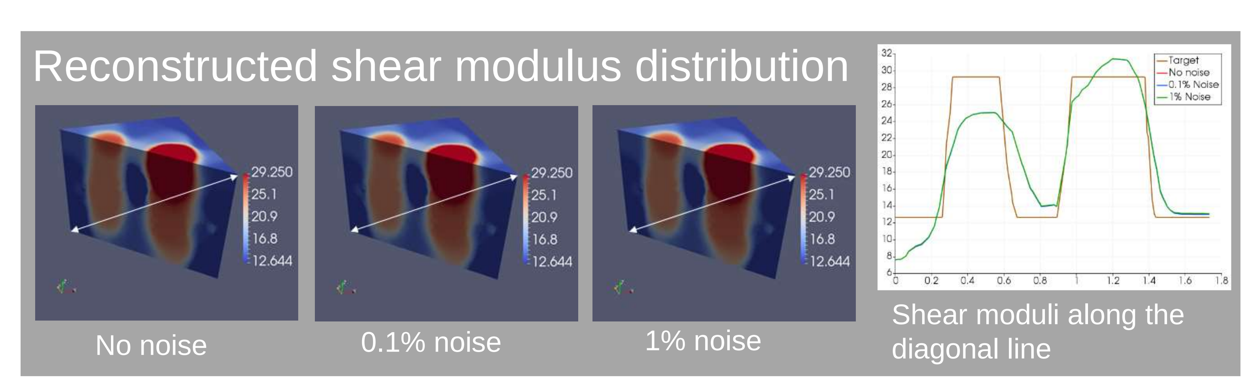
SIMULATIONS WITH DEVIATION OF FORCE LOCATIONS



- Boundary conditions : fix the bottom surface; Apply force boundary conditions to the left and right surfaces, and front and back surfaces.
- Simulate 10 boundary value problems to create 10 displacement fields.
- Minimize displacements on five surfaces: front, back, left, right, and top.
- The center of the forces deviates 0.4421 mm, a 4.653% magnitude relative to the radius of the force disk.



- The center of the forces deviates 1.7682 mm, a 18.612% magnitude relative to the radius of the force disk.



CONCLUSION

- Shear moduli can be correctly reconstructed with no prior assumption about the material property using our in-house inverse solver.
- More data sets result in more accurate reconstructions.
- Location of the force indentations affect their effectiveness of identifying the existence of an inclusion.
- Uncertainty in the force locations affects shear modulus reconstruction more than noise in displacements.

REFERENCE

Mei, Y., Fulmer, R., Raja, V., Wang, S. and Goenezen, S., 2016. Estimating the non-homogeneous elastic modulus distribution from surface deformations. International Journal of Solids and Structures, 83, pp. 73-80.

Mei, Y., Wang, S., Shen, X., Rabke, S., Goenezen, S., 2017. Mechanics based tomography: A Preliminary feasibility study . Sensors, 17(5), pp. 1075.

Luo, P., Mei, Y., Kotecha, M., Abbasszadehrad A., Rabke, S., Garner, G., Goenezen, S., 2018. Characterization of the Stiffness Distribution in 2D and 3D. MRS Communications, doi:10.1557/mrc.2018.98